Course Outlines

https://courseoutline.auckland.ac.nz/dco/course/MATHS/714/1215

MATHS 714 : Number Theory

Science

2021 Semester Two (1215) (15 POINTS)

Course Prescription

A broad introduction to various aspects of elementary, algebraic and computational number theory and its applications, including primality testing and cryptography.

Course Overview

Number theory is one of the oldest and most beautiful branches of mathematics. It concerns problems involving prime numbers and Diophantine equations, and includes many of the most famous problems in mathematics such as Fermat's last theorem and the Goldbach conjecture. Number theory continues to be a dynamic research area with numerous connections with other branches of mathematics as well as applications in the real-world (such as public key cryptography). The course builds on undergraduate knowledge in pure mathematics (especially abstract algebra, as seen in MATHS 320 and MATHS 328). The course gives an overview of basic methods and results in number theory.

Course Requirements

Prerequisite: B+ in MATHS 328 or 320

Capabilities Developed in this Course

Capability 1:   Disciplinary Knowledge and Practice
Capability 2:   Critical Thinking
Capability 3:   Solution Seeking
Capability 4:   Communication and Engagement
Capability 5:   Independence and Integrity

Graduate Profile: Bachelor of Science (Honours)

Learning Outcomes

By the end of this course, students will be able to:

1. Demonstrate familiarity and confidence in using the definitions and tools of number theory, and ability to solve challenging theoretical problems in number theory (Capability 1, 2 and 3)
2. Demonstrate ability to write clear and precise proofs and to communicate effectively in written form (Capability 4 and 5)
3. Demonstrate ability to solve linear congruences, use the Chinese Remainder Theorem, compute with primitive roots and multiplicative functions, and prove existence of primitive roots modulo p (Capability 1)
4. Demonstrate ability to compute Legendre symbols and to prove the quadratic reciprocity law (Capability 1 and 4)
5. Demonstrate understanding of Dirichlet's theorem, and ability to prove infinitely many primes in residue classes in some special cases (Capability 2, 3 and 4)
6. Demonstrate ability to compute continued fractions and to solve Pell-type equations (Capability 1, 3 and 5)
7. Demonstrate ability to compute with elliptic curves and to determine points of finite order and the rank of elliptic curves in special cases (Capability 1, 3 and 5)
8. Demonstrate understanding of algebraic numbers and, in particular, the class group and the class number (Capability 1 and 4)
9. Demonstrate ability to use connections with algebraic number theory to solve certain classes of Diophantine equations (Capability 3 and 5)

Assessments

Assessment Type	Percentage	Classification
Assignments	40%	Individual Coursework
Test	10%	Individual Test
Final Exam	50%	Individual Examination
3 types	100%	

Assessment Type	Learning Outcome Addressed								
	1	2	3	4	5	6	7	8	9
Assignments	✔	✔	✔	✔	✔	✔	✔	✔	✔
Test	✔	✔	✔	✔	✔	✔	✔	✔	✔
Final Exam	✔	✔	✔	✔	✔	✔	✔	✔	✔

Plussage:

A student's final mark (out of 100) will be determined by the maximum of the following two options:

1) 40 marks from Assignments, 10 marks from the Test, 50 marks from the Final Exam

2) 100 marks from the Final Exam.

Every student needs to score at least 35% in the Final Exam to pass the course.

Tuākana

No additional Tuākana support for this course

Key Topics

Divisibility, primes, congruences, Euclid algorithm, Chinese remainder theorem.

Multiplicative functions, primitive roots and the group structure of Z_n^*.

Legendre symbols and quadratic reciprocity.

Prime number theorem and Dirichlet's theorem.

Continued fractions and Pell's equation.

Elliptic curves.

Algebraic number theory, Galois theory, and geometry of numbers.

Special Requirements

None

Workload Expectations

This course is a standard 15 point course and students are expected to spend 10 hours per week involved in each 15 point course that they are enrolled in.

For this course, you can expect 3 hours of lectures, 3 hours of reading and thinking about the content and 4 hours of work on assignments and/or preparation for the test or exam.

Delivery Mode

Campus Experience

Activities for the course are scheduled in a standard weekly timetable. Attendance is recommended at lectures, which will not be available as recordings (unless the campus is closed and lectures have to be delivered online). There are no tutorials or group discussions. Attendance on campus is required for the Test (unless the campus is closed and the Test has to be undertaken online). Attendance on campus is required for the Final Exam (unless the campus is closed and the Final Exam has to be undertaken online).

Learning Resources

Lecture notes are provided on Canvas. There is an extensive Talis reading list, but there is no set coursebook.

Student Feedback

During the course Class Representatives in each class can take feedback to the staff responsible for the course and staff-student consultative committees.

At the end of the course students will be invited to give feedback on the course and teaching through a tool called SET or Qualtrics. The lecturers and course co-ordinators will consider all feedback.

Your feedback helps to improve the course and its delivery for all students.

Other Information

None

Digital Resources

Course materials are made available in a learning and collaboration tool called Canvas which also includes reading lists and lecture recordings (where available).

Please remember that the recording of any class on a personal device requires the permission of the instructor.

Academic Integrity

The University of Auckland will not tolerate cheating, or assisting others to cheat, and views cheating in coursework as a serious academic offence. The work that a student submits for grading must be the student's own work, reflecting their learning. Where work from other sources is used, it must be properly acknowledged and referenced. This requirement also applies to sources on the internet. A student's assessed work may be reviewed against online source material using computerised detection mechanisms.

Copyright

The content and delivery of content in this course are protected by copyright. Material belonging to others may have been used in this course and copied by and solely for the educational purposes of the University under license.

You may copy the course content for the purposes of private study or research, but you may not upload onto any third party site, make a further copy or sell, alter or further reproduce or distribute any part of the course content to another person.

Inclusive Learning

All students are asked to discuss any impairment related requirements privately, face to face and/or in written form with the course coordinator, lecturer or tutor.

Student Disability Services also provides support for students with a wide range of impairments, both visible and invisible, to succeed and excel at the University. For more information and contact details, please visit the Student Disability Services' website http://disability.auckland.ac.nz

Special Circumstances

If your ability to complete assessed coursework is affected by illness or other personal circumstances outside of your control, contact a member of teaching staff as soon as possible before the assessment is due.

If your personal circumstances significantly affect your performance, or preparation, for an exam or eligible written test, refer to the University's aegrotat or compassionate consideration page https://www.auckland.ac.nz/en/students/academic-information/exams-and-final-results/during-exams/aegrotat-and-compassionate-consideration.html.

This should be done as soon as possible and no later than seven days after the affected test or exam date.

Learning Continuity

In the event of an unexpected disruption we undertake to maintain the continuity and standard of teaching and learning in all your courses throughout the year. If there are unexpected disruptions the University has contingency plans to ensure that access to your course continues and your assessment is fair, and not compromised. Some adjustments may need to be made in emergencies. You will be kept fully informed by your course co-ordinator, and if disruption occurs you should refer to the University Website for information about how to proceed.

The following apply with regard to Covid Alert Levels:
Level 1: Delivered normally as specified in delivery mode.
Level 2: You will not be required to attend in person. All teaching and assessment will have a remote option.
Level 3 or 4: All teaching activities and assessments are delivered remotely.

Student Charter and Responsibilities

The Student Charter assumes and acknowledges that students are active participants in the learning process and that they have responsibilities to the institution and the international community of scholars. The University expects that students will act at all times in a way that demonstrates respect for the rights of other students and staff so that the learning environment is both safe and productive. For further information visit Student Charter https://www.auckland.ac.nz/en/students/forms-policies-and-guidelines/student-policies-and-guidelines/student-charter.html.

Disclaimer

Elements of this outline may be subject to change. The latest information about the course will be available for enrolled students in Canvas.

In this course you may be asked to submit your coursework assessments digitally. The University reserves the right to conduct scheduled tests and examinations for this course online or through the use of computers or other electronic devices. Where tests or examinations are conducted online remote invigilation arrangements

may be used. The final decision on the completion mode for a test or examination, and remote invigilation arrangements where applicable, will be advised to students at least 10 days prior to the scheduled date of the assessment, or in the case of an examination when the examination timetable is published.