Course Outlines

https://courseoutline.auckland.ac.nz/dco/course/MATHS/253/1223

MATHS 253 : Algebra and Calculus 3

Science

2022 Semester One (1223) (15 POINTS)

Course Prescription

A sequel to MATHS 250, further developing and bringing together linear algebra and calculus. Students will learn about quadratic forms, projections, spectral decomposition, methods of multicriteria optimisation, double, triple and line integrals, Green's theorem and applications.

Course Overview

This course covers topics in linear algebra, multi-variable calculus and their applications. In Linear Algebra students learn about normal forms of operators, in particular, Jordan normal form, diagonalisation of Hermitian operators, polynomial and Fourier approximations and quadratic forms. In Calculus, students learn partial derivatives and the gradient, methods of optimisation of functions of several variables, double, triple and line integrals, conservative vector fields, and, finally, an all-important Green's theorem. The course shows unity and inseparability of Algebra and Calculus. It lays a foundation for a large number of stage 3 and graduate courses in pure and applied mathematics, statistics, and also for many advanced courses in physics and various other applied sciences. It is a core mathematics course for the BSc(Hons), BAdvSci and PGDipSci.

Course Requirements

Prerequisite: MATHS 250

Capabilities Developed in this Course

Capability 1: Disciplinary Knowledge and Practice
Capability 2: Critical Thinking
Capability 3: Solution Seeking
Capability 4: Communication and Engagement
Capability 5: Independence and Integrity
Capability 6: Social and Environmental Responsibilities

Graduate Profile: Bachelor of Science

Learning Outcomes

By the end of this course, students will be able to:

1. Display a high level of knowledge of linear algebra and calculus (Capability 1 and 3)
2. Demonstrate an understanding of mutual penetration and inseparability of algebra and calculus at the highest level of mathematics (Capability 2 and 4)
3. Be able to solve problems that require knowledge of both algebraic and calculus ideas (Capability 1, 2 and 3)
4. Be prepared to work in teams, critically discuss various approaches to modelling and solution of complex applied problems (Capability 3, 4 and 5)
5. Engage in group discussions and critical interactions about the role of mathematics in the society (Capability 2, 4, 5 and 6)

Assessments

Assessment Type	Percentage	Classification
Assignments	24%	Individual Coursework
Tutorials (combined with quizzes)	6%	Individual Coursework
Test	25%	Individual Test
Final Exam	45%	Individual Examination
4 types	100%	

Assessment Type	Learning Outcome Addressed				
	1	2	3	4	5
Assignments	✔	✔	✔		
Tutorials (combined with quizzes)		✔		✔	✔
Test	✔	✔	✔		
Final Exam	✔	✔	✔		

Key Topics

Linear Algebra

1. (Mostly revision) Vector spaces over R and C and their subspaces. Bases & dimension. The coordinate mapping.

2. Change of basis matrices and their properties. Linear transformations and their matrices.
3. Matrices of linear operators in various bases. Algebra of linear operators.
4. Diagonalisation of operators. Criteria of diagonalisability. Invariant subspaces.
5. Applications of diagonalisation. Discrete time system evolution.
6. Fitting lemma. Caley-Hamilton theorem.
7. Root spaces and root space decomposition of an operator.
8. Jordan normal form of an operator.
9. Inner products and real inner product spaces. Orthogonality in inner product spaces and their orthogonal bases.
10. Projections as best approximations. Polynomial and Fourier approximations.
11. Complex inner products. Orthogonal matrices.
12. Adjoint operator. The normal equation.
12. Orthogonal diagonalisation of Hermitian operators on C^n.
13. Orthogonal diagonalisation of Hermitian operators on R^n and symmetric matrices. Spectral decomposition.
14. Quadratic forms and their matrices. Change of basis. Principal axes theorem.
15. Conics and Quadrics.
16. Positive definite quadratic forms. Necessity of Sylvester's criterion.
17. Congruent diagonalisation. Sufficiency of Sylvester's criterion (time permitting).

Calculus

18. Partial derivatives. Higher order partial derivatives. Symmetry of the Hessian matrix.
19. First order approximations. Differentiability.
20. The chain rule and applications.
21. Gradient, tangent planes, directional derivatives.
22. Taylor series. Best quadratic approximation.
23. Maxima and minima. Critical points.
24. Constrained and unconstrained optimisation.
25. Double integrals over rectangles.
26. Fubini's theorem.
27. Double integrals over general domains.
28. Change of variables in double integrals.
29. Triple integrals.
30. Space curves. Arc length parametrisation.
31. Surfaces and their areas.
32. Vector fields. Conservative vector fields.
33. Line integrals.
34. Green's theorem.

Special Requirements

There are no special requirements in this course.

Workload Expectations

This course is a standard 15 point course and students are expected to spend 10 hours per week involved in each 15 point course that they are enrolled in.

For this course, you can expect 36 hours of lectures, a 10 hour tutorials, 36 hours of reading and thinking about the content and 40 hours of work on assignments and/or test preparation.

Delivery Mode

Campus Experience

Attendance is expected at scheduled activities including tutorials. Lectures will be available as recordings. Online tutorials for overseas students will be available.
The course will not include live online events.
Attendance on campus is required for the test and exam.
The activities for the course are scheduled as a standard weekly timetable.

Learning Resources

The course book for this course is available to students through SRC or electronically from Canvas. The text books are Linear Algebra, (Poole) and Calculus, (Stewart), any edition. They are available at the University Bookshop.

Student Feedback

During the course Class Representatives in each class can take feedback to the staff responsible for the course and staff-student consultative committees.

At the end of the course students will be invited to give feedback on the course and teaching through a tool called SET or Qualtrics. The lecturers and course co-ordinators will consider all feedback.

Your feedback helps to improve the course and its delivery for all students.

Academic Integrity

The University of Auckland will not tolerate cheating, or assisting others to cheat, and views cheating in coursework as a serious academic offence. The work that a student submits for grading must be the student's own work, reflecting their learning. Where work from other sources is used, it must be properly acknowledged and referenced. This requirement also applies to sources on the internet. A student's assessed work may be reviewed against online source material using computerised detection mechanisms.

Class Representatives

Class representatives are students tasked with representing student issues to departments, faculties, and the wider university. If you have a complaint about this course, please contact your class rep who will know how to raise it in the right channels. See your departmental noticeboard for contact details for your class reps.

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Inclusive Learning

All students are asked to discuss any impairment related requirements privately, face to face and/or in written form with the course coordinator, lecturer or tutor.

Student Disability Services also provides support for students with a wide range of impairments, both visible and invisible, to succeed and excel at the University. For more information and contact details, please visit the Student Disability Services' website http://disability.auckland.ac.nz

Special Circumstances

If your ability to complete assessed coursework is affected by illness or other personal circumstances outside of your control, contact a member of teaching staff as soon as possible before the assessment is due.

If your personal circumstances significantly affect your performance, or preparation, for an exam or eligible written test, refer to the University's aegrotat or compassionate consideration page https://www.auckland.ac.nz/en/students/academic-information/exams-and-final-results/during-exams/aegrotat-and-compassionate-consideration.html.

This should be done as soon as possible and no later than seven days after the affected test or exam date.

Learning Continuity

In the event of an unexpected disruption, we undertake to maintain the continuity and standard of teaching and learning in all your courses throughout the year. If there are unexpected disruptions the University has contingency plans to ensure that access to your course continues and course assessment continues to meet the principles of the University's assessment policy. Some adjustments may need to be made in emergencies. You will be kept fully informed by your course co-ordinator/director, and if disruption occurs you should refer to the university website for information about how to proceed.

The delivery mode may change depending on COVID restrictions. Any changes will be communicated through Canvas.

Student Charter and Responsibilities

The Student Charter assumes and acknowledges that students are active participants in the learning process and that they have responsibilities to the institution and the international community of scholars. The University expects that students will act at all times in a way that demonstrates respect for the rights of other students and staff so that the learning environment is both safe and productive. For further information visit Student Charter https://www.auckland.ac.nz/en/students/forms-policies-and-guidelines/student-policies-and-guidelines/student-charter.html.

Disclaimer

Elements of this outline may be subject to change. The latest information about the course will be available for enrolled students in Canvas.

In this course students may be asked to submit coursework assessments digitally. The University reserves the right to conduct scheduled tests and examinations for this course online or through the use of computers or other electronic devices. Where tests or examinations are conducted online remote invigilation arrangements may be used. In exceptional circumstances changes to elements of this course may be necessary at short notice. Students enrolled in this course will be informed of any such changes and the reasons for them, as soon as possible, through Canvas.